Chapter 13 - Elasticity

A PowerPoint Presentation by

Paul E. Tippens, Professor of Physics

Southern Polytechnic State University

Chapter 13. Elasticity

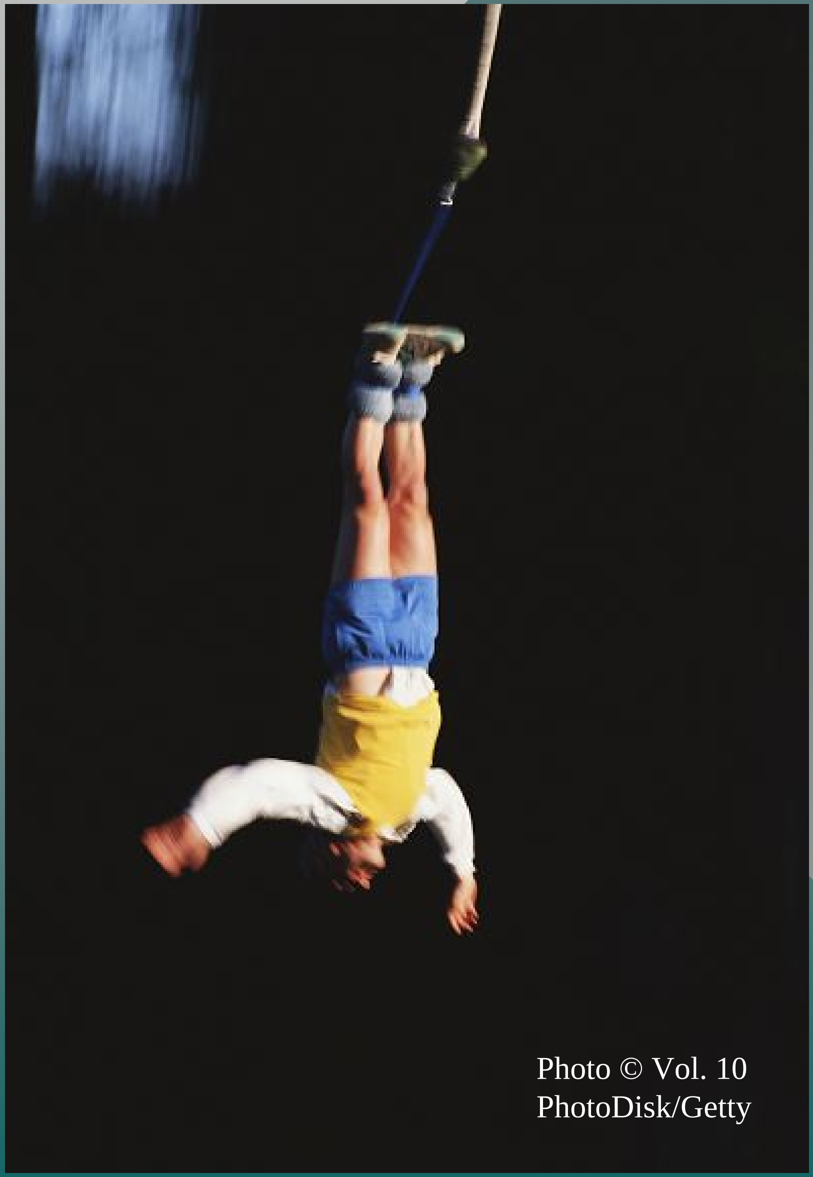


Photo © Vol. 10 PhotoDisk/Getty

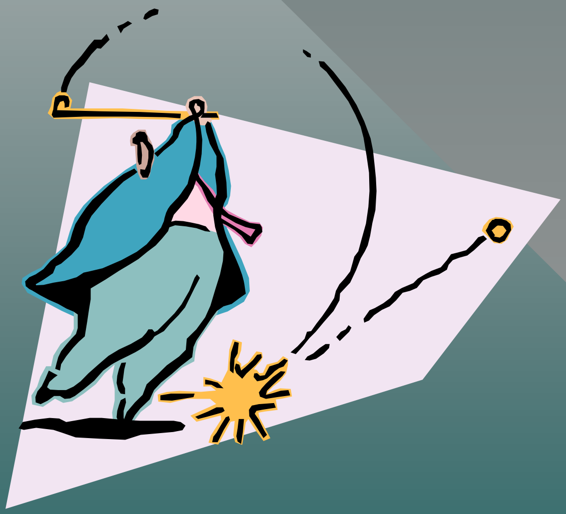
BUNGEE jumping utilizes a long elastic strap which stretches until it reaches a maximum length that is proportional to the weight of the jumper. The elasticity of the strap determines the amplitude of the resulting vibrations. If the *elastic limit* for the strap is exceeded, the rope will break.

Objectives: After completion of this module, you should be able to:

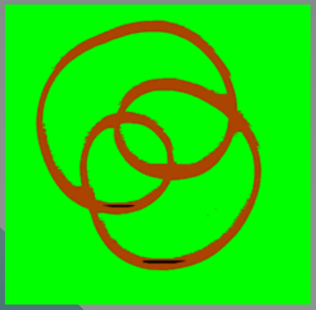
- Demonstrate your understanding of *elasticity*, *elastic limit*, *stress*, *strain*, *and ultimate strength*.
- Write and apply formulas for calculating Young's modulus, shear modulus, and bulk modulus.
- Solve problems involving each of the parameters in the above objectives.

Elastic Properties of Matter

An **elastic body** is one that returns to its original shape after a deformation.



Golf Ball



Rubber Band

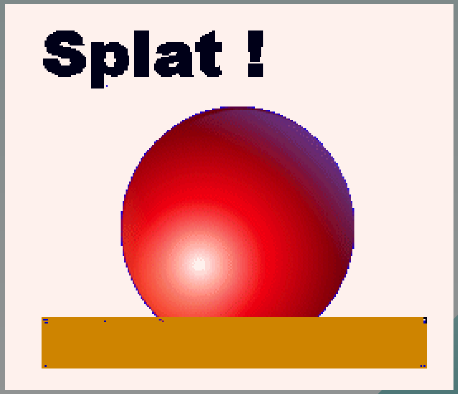
Soccer Ball

Elastic Properties of Matter

An **inelastic body** is one that does **not** return to its original shape after a deformation.

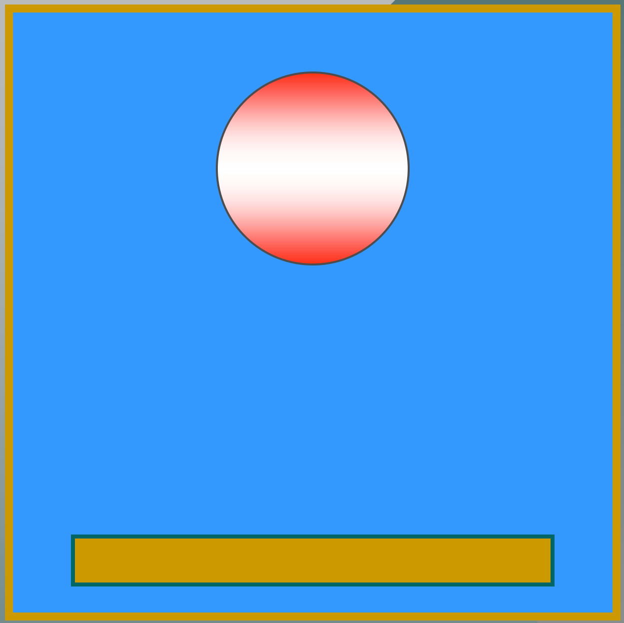
Dough or Bread

Clay

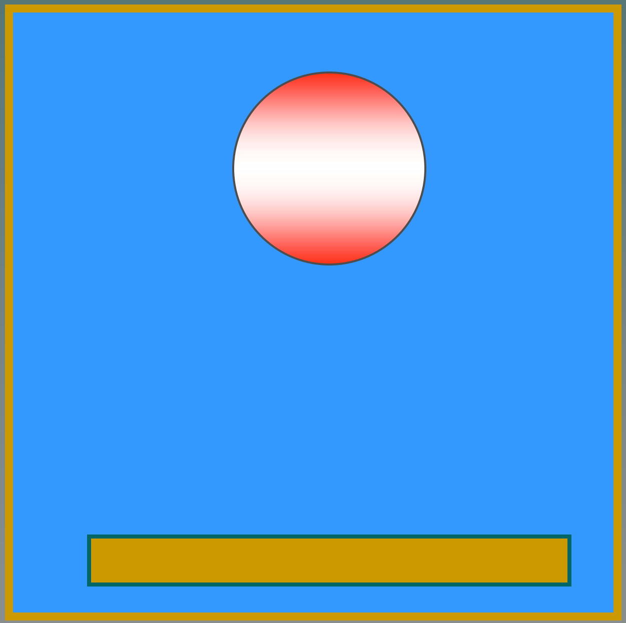


Inelastic Ball

Elastic or Inelastic?



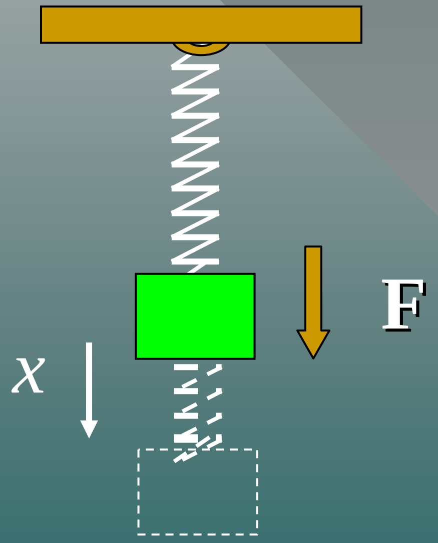
An elastic collision loses no energy. The deform-ation on collision is fully restored.



In an inelastic collision, energy is lost and the deformation may be permanent. (Click it.)

An Elastic Spring

A spring is an example of an elastic body that can be deformed by stretching.

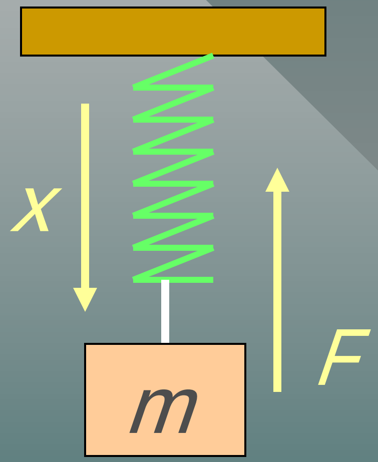


A **restoring force**, ***F***, acts in the direction opposite the displacement of the oscillating body.

$$F = -kx$$

Hooke's Law

When a spring is stretched, there is a restoring force that is proportional to the displacement.



$$F = -kx$$

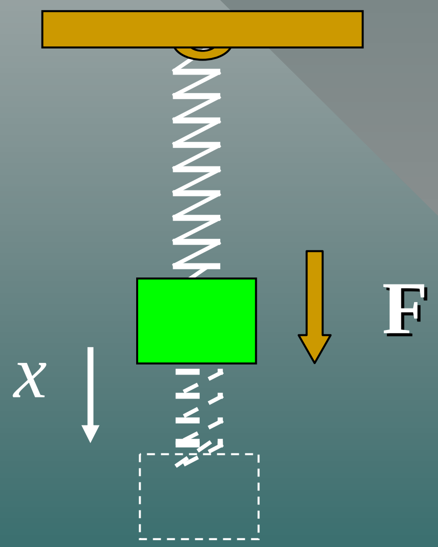
The spring constant k is a property of the spring given by:

$$k = \frac{\Delta F}{\Delta x}$$

The spring constant k is a measure of the elasticity of the spring.

Stress and Strain

Stress refers to the cause of a deformation, and strain refers to the effect of the deformation.

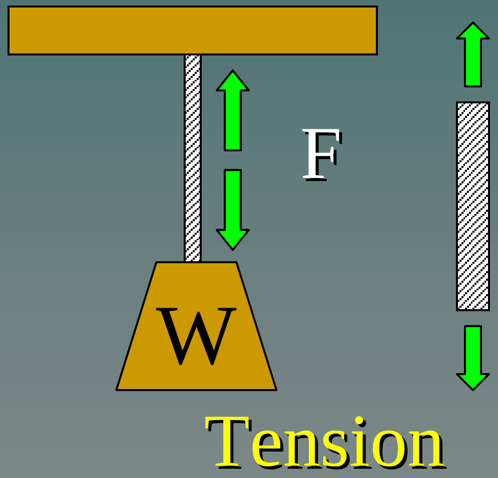


The downward force F causes the displacement x.

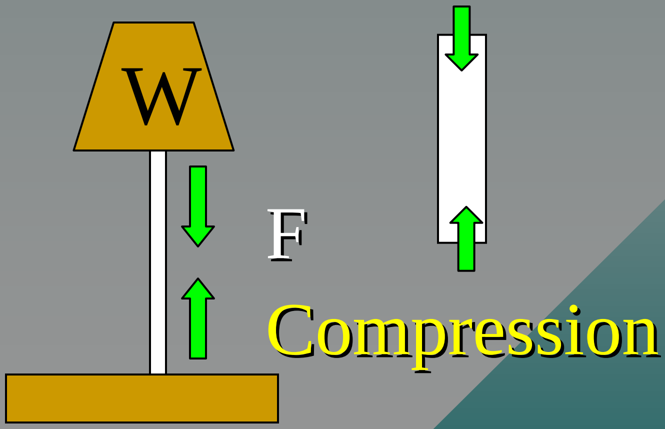
Thus, the stress is the force; the strain is the elongation.

Types of Stress

A tensile stress occurs when equal and opposite forces are directed away from each other.



A compressive stress occurs when equal and opposite forces are directed toward each other.



Summary of Definitions

Stress is the ratio of an applied force F to the area A over which it acts:

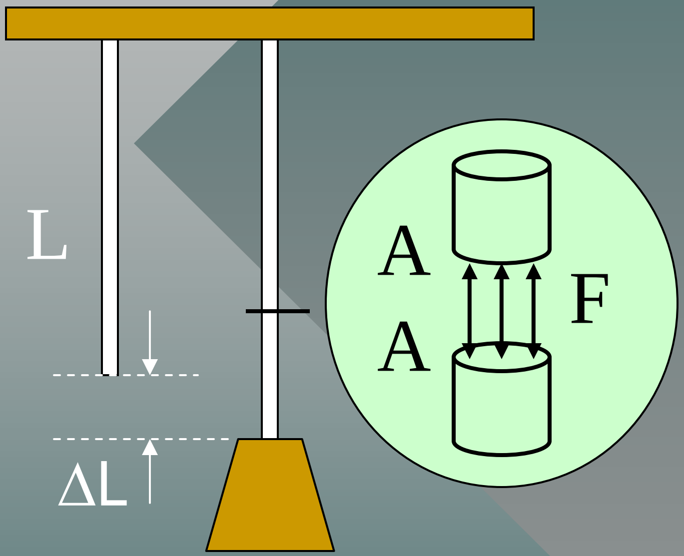
$$\text{Stress} = \frac{F}{A}$$

$$\text{Units: } \text{Pa} = \frac{\text{N}}{\text{m}^2} \text{ or } \frac{\text{lb}}{\text{in.}^2}$$

Strain is the relative change in the dimensions or shape of a body as the result of an applied stress:

Examples: Change in length per unit length;
change in volume per unit volume.

Longitudinal Stress and Strain

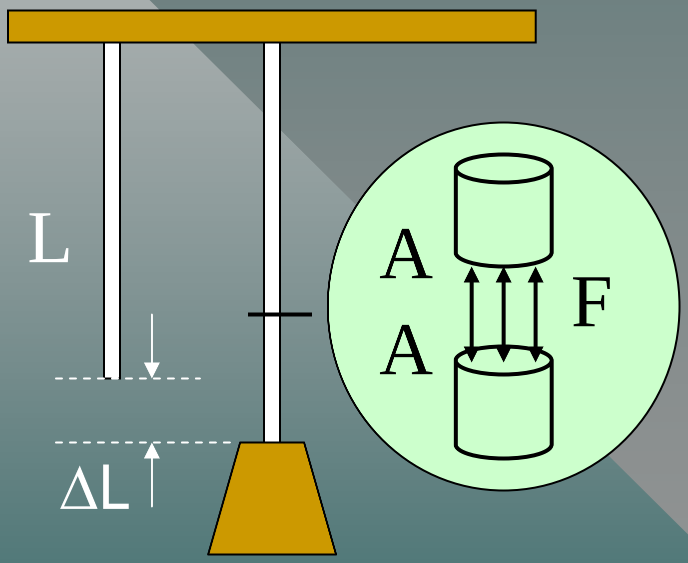


For wires, rods, and bars, there is a longitudinal stress F/A that produces a change in length per unit length. In such cases:

$$Stress = \frac{F}{A}$$

$$Strain = \frac{\Delta L}{L}$$

Example 1. A steel wire 10 m long and 2 mm in diameter is attached to the ceiling and a 200-N weight is attached to the end. What is the applied stress?



First find area of wire:

$$A = \frac{\pi D^2}{4} = \frac{\pi (0.002\text{ m})^2}{4}$$

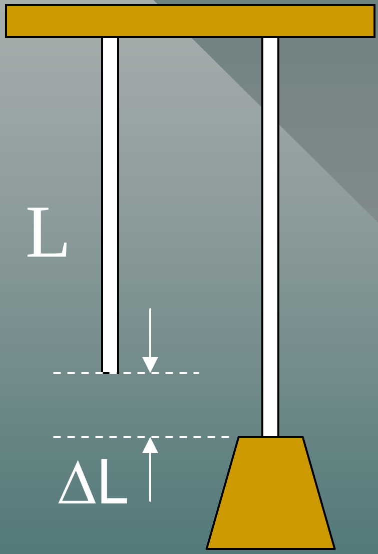
$$A = 3.14 \text{ x } 10^{-6}\text{ m}^2$$

$$Stress = \frac{F}{A} = \frac{200\text{ N}}{3.14 \text{ x } 10^{-6}\text{ m}^2}$$

Stress

$6.37 \text{ x } 10^7$ Pa

Example 1 (Cont.) A 10 m steel wire stretches 3.08 mm due to the 200 N load. What is the longitudinal strain?



Given: L = 10 m; ΔL = 3.08 mm

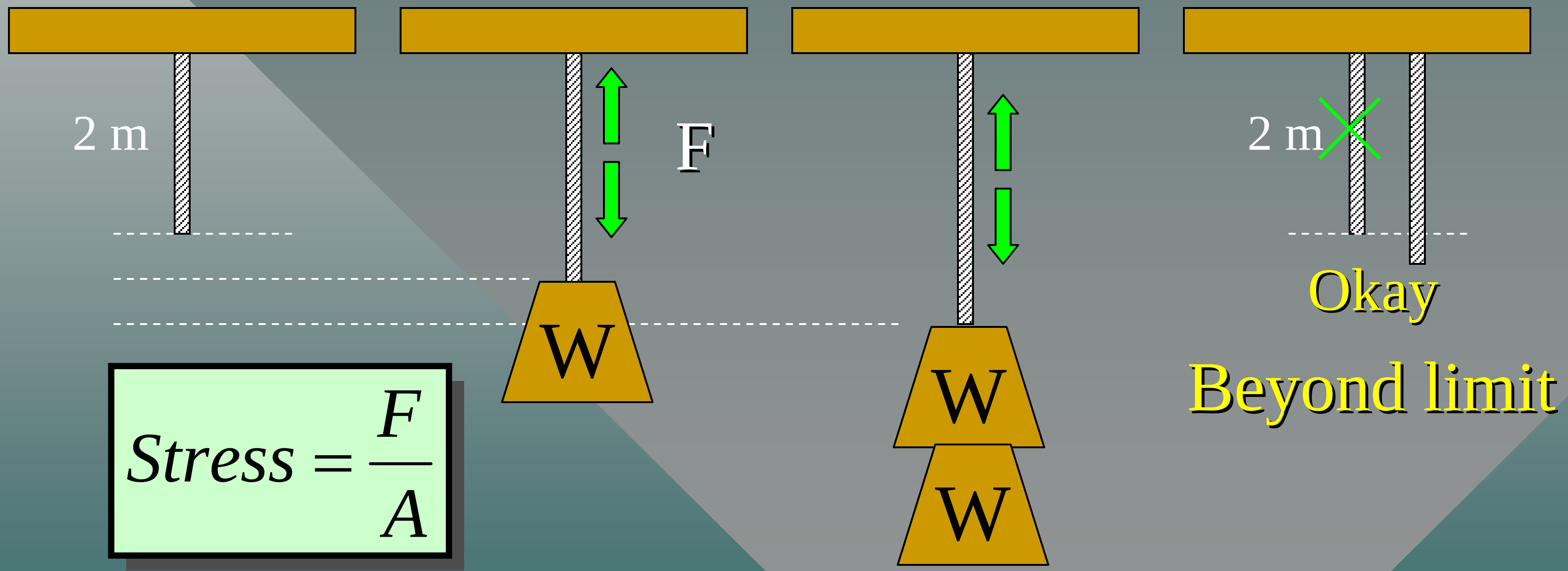
$$Srain = \frac{\Delta L}{L} = \frac{0.00308\ \text{m}}{10\ \text{m}}$$

Longitudinal Strain

3.08×10^{-4}

The Elastic Limit

The elastic limit is the maximum stress a body can experience without becoming permanently deformed.

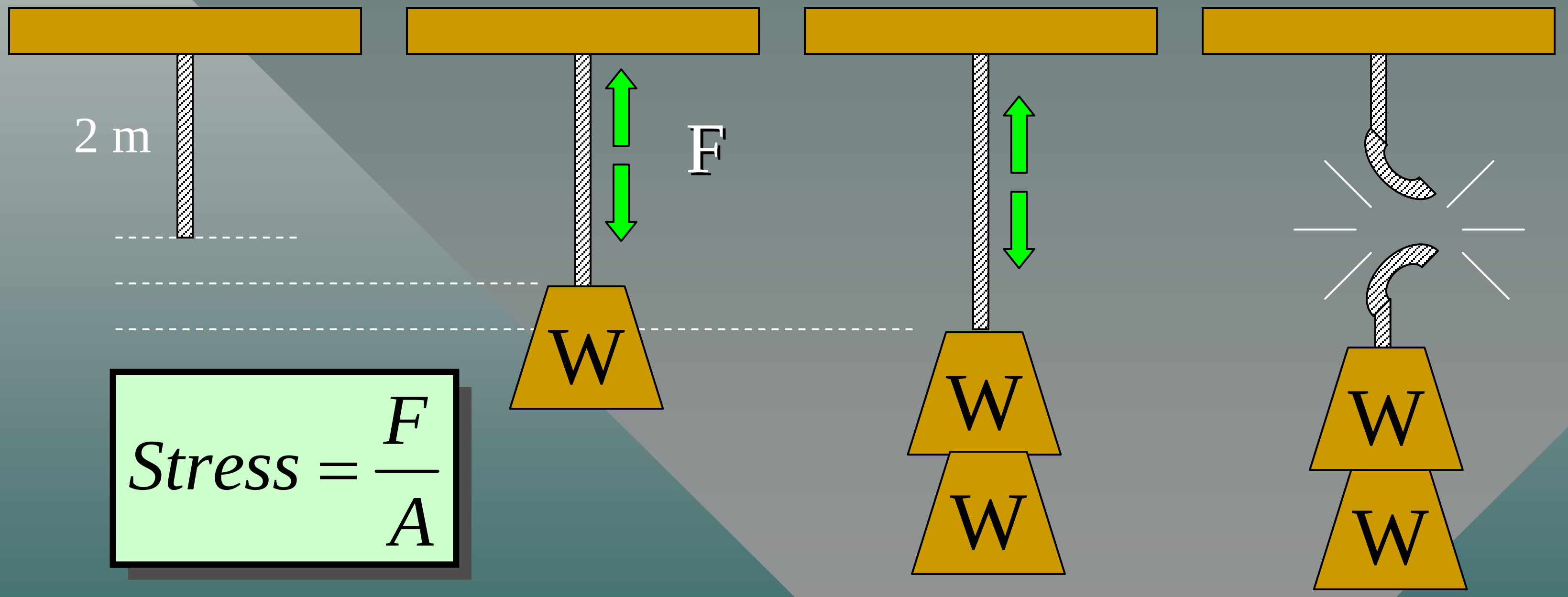


$$Stress = \frac{F}{A}$$

If the stress exceeds the elastic limit, the final length will be longer than the original 2 m.

The Ultimate Strength

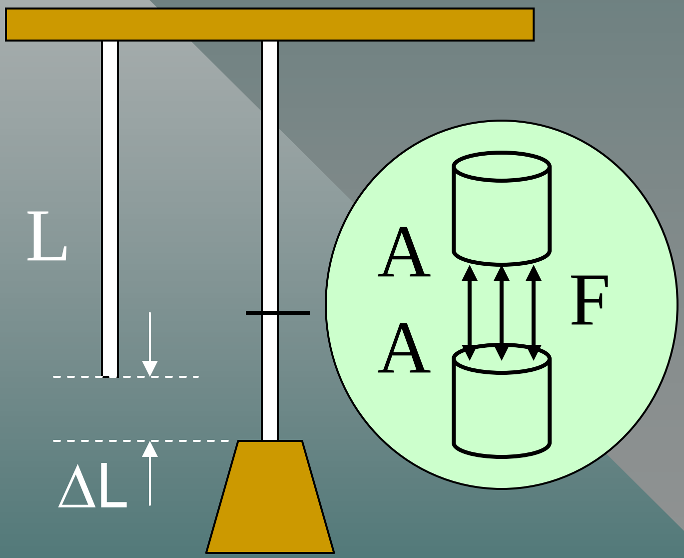
The ultimate strength is the greatest stress a body can experience without breaking or rupturing.



$$Stress = \frac{F}{A}$$

If the stress exceeds the ultimate strength, the string breaks!

Example 2. The elastic limit for steel is 2.48×10^8 Pa. What is the maximum weight that can be supported without exceeding the elastic limit?



Recall: $A = 3.14 \times 10^{-6}\ \text{m}^2$

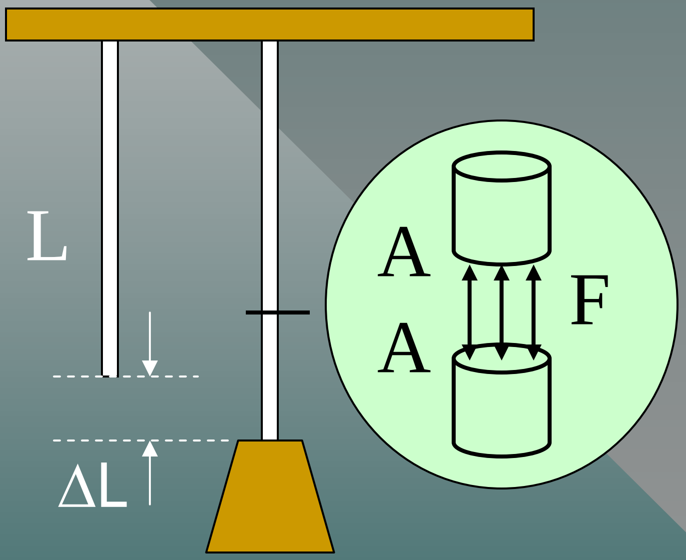
$$Stress = \frac{F}{A} = 2.48 \times 10^8\ \text{Pa}$$

$$F = (2.48 \times 10^8\ \text{Pa})\,A$$

$$F = (2.48 \times 10^8\ \text{Pa})(3.14 \times 10^{-6}\ \text{m}^2)$$

$$F = 779\ \text{N}$$

Example 2(Cont.) The ultimate strength for steel is 4089 x 10^8 Pa. What is the maxi- mum weight that can be supported without breaking the wire?



Recall: $A = 3.14 \text{ x } 10^{-6} \text{ m}^2$

$$Stress = \frac{F}{A} = 4.89 \text{ x } 10^8 \text{ Pa}$$

$$F = (4.89 \text{ x } 10^8 \text{ Pa})\, A$$

$$F = (4.89 \text{ x } 10^8 \text{ Pa})(3.14 \text{ x } 10^{-6} \text{ m}^2)$$

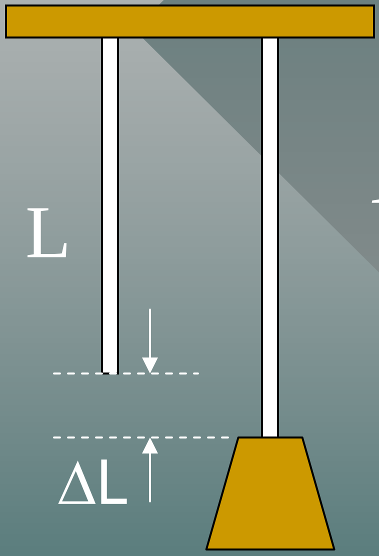
$$F = 1536 \text{ N}$$

The Modulus of Elasticity

Provided that the elastic limit is not exceeded, an elastic deformation (strain) is directly proportional to the magnitude of the applied force per unit area (stress).

$$Modulus\ of\ \ Elasticity = \frac{stress}{strain}$$

Example 3. In our previous example, the stress applied to the steel wire was 6.37×10^7 Pa and the strain was 3.08×10^{-4}. Find the modulus of elasticity for steel.



$$Modulus = \frac{Stress}{Strain} = \frac{6.37 \times 10^7 \text{ Pa}}{3.08 \times 10^{-4}}$$

Modulus = 207×10^9 Pa

This longitudinal modulus of elasticity is called Young's Modulus and is denoted by the symbol *Y*.

Young's Modulus

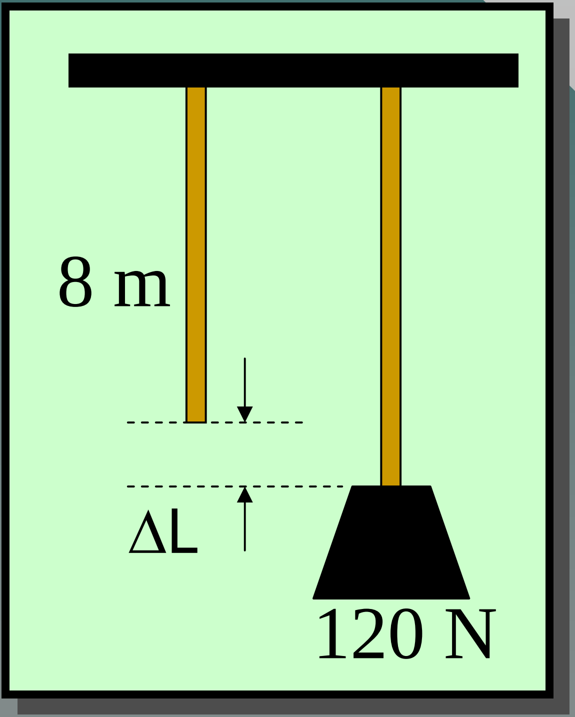
For materials whose length is much greater than the width or thickness, we are concerned with the longitudinal modulus of elasticity, or Young's Modulus (Y).

$$Young's\ modulus = \frac{longitudinal\ stress}{longitudinal\ strain}$$

$$Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

$$Units:\ \text{Pa}\ \ or\ \ \frac{\text{lb}}{\text{in.}^2}$$

Example 4: Young's modulus for brass is 8.96 x 10^{11}Pa. A 120-N weight is attached to an 8-m length of brass wire; find the increase in length. The diameter is 1.5 mm.



First find area of wire:

$$A = \frac{\pi D^2}{4} = \frac{\pi (0.0015\text{ m})^2}{4}$$

A = 1.77 x 10^{-6} m^2

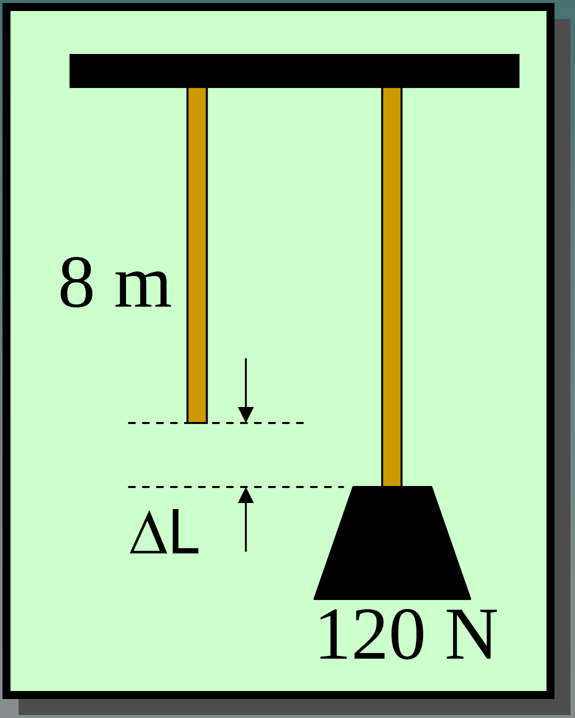
$$Y = \frac{FL}{A\Delta L} \quad \text{or} \quad \Delta L = \frac{FL}{AY}$$

Example 4: (Continued)

$Y = 8.96 \times 10^{11}$ Pa; $F = 120$ N;

$L = 8$ m; $A = 1.77 \times 10^{-6}$ m^2

$F = 120$ N; $\Delta L = ?$



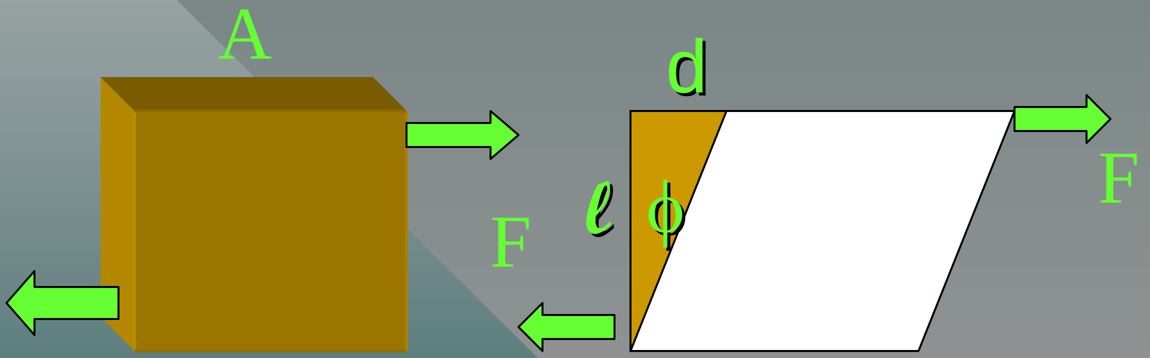
$$Y = \frac{FL}{A\Delta L} \quad \text{or} \quad \Delta L = \frac{FL}{AY}$$

$$\Delta L = \frac{FL}{AY} = \frac{(120 \text{ N})(8.00 \text{ m})}{(1.77 \times 10^{-6} \text{m}^2)(8.96 \times 10^{11} \text{Pa})}$$

Increase in length: $\Delta L = 0.605$ mm

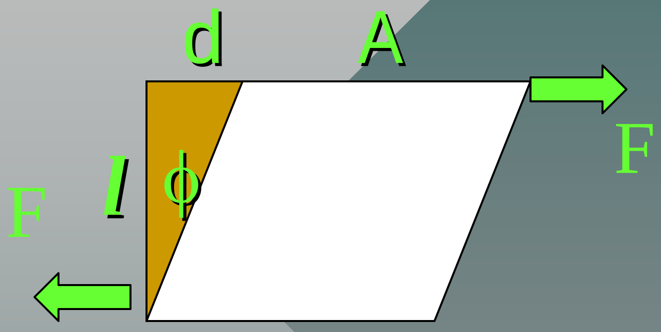
Shear Modulus

A shearing stress alters only the shape of the body, leaving the volume unchanged. For example, consider equal and opposite shearing forces F acting on the cube below:



The shearing force F produces a shearing angle ϕ. The angle ϕ is the strain and the stress is given by F/A as before.

Calculating Shear Modulus



Stress is force per unit area:

$$Stress = \frac{F}{A}$$

The strain is the angle expressed in radians:

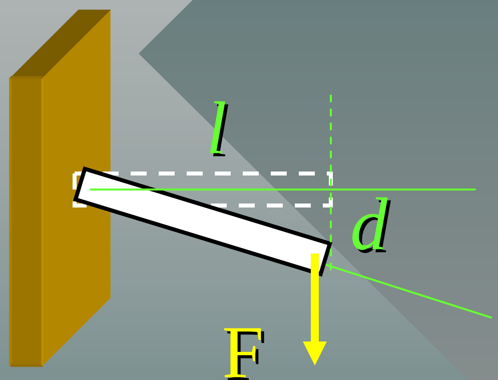
$$Strain = \phi = \frac{d}{l}$$

The shear modulus S is defined as the ratio of the shearing stress *F/A* to the shearing strain ϕ:

The shear modulus:
Units are in Pascals.

$$S = \frac{F/A}{\phi}$$

Example 5. A steel stud ($S = 8.27 \times 10^{10}$ Pa) 1 cm in diameter projects 4 cm from the wall. A 36,000 N shearing force is applied to the end. What is the defection *d* of the stud?



$$A = \frac{\pi D^2}{4} = \frac{\pi (0.01\text{ m})^2}{4}$$

Area: $A = 7.85 \times 10^{-5}\text{ m}^2$

$$S = \frac{F/A}{\phi} = \frac{F/A}{d/l} = \frac{Fl}{Ad}; \qquad d = \frac{Fl}{AS}$$

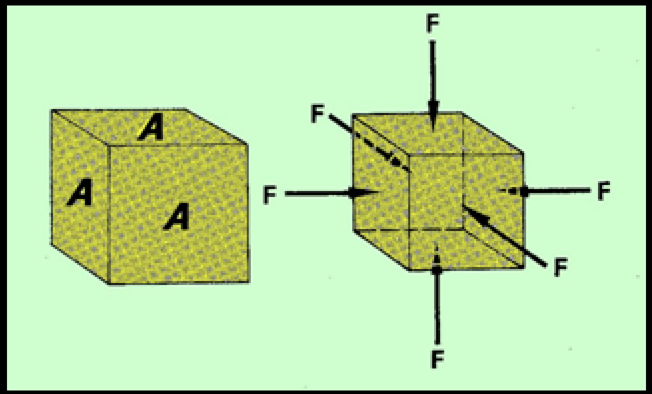
$$d = \frac{(36{,}000\text{ N})(0.04\text{ m})}{(7.85 \times 10^{-5}\text{m}^2)(8.27 \times 10^{10}\text{Pa})}$$

***d* = 0.222 mm**

Volume Elasticity

Not all deformations are linear. Sometimes an applied stress F/A results in a decrease of volume. In such cases, there is a bulk modulus B of elasticity.

$$B = \frac{\text{Volume stress}}{\text{Volume strain}} = \frac{-F/A}{\Delta V/V}$$



The bulk modulus is negative because of decrease in V.

The Bulk Modulus

$$B = \frac{\text{Volume stress}}{\text{Volume strain}} = \frac{-F/A}{\Delta V/V}$$

Since *F/A* is generally pressure *P*, we may write:

$$B = \frac{-P}{\Delta V/V} = \frac{-PV}{\Delta V}$$

Units remain in Pascals (Pa) since the strain is unitless.

Example 7. A hydrostatic press contains 5 liters of oil. Find the decrease in volume of the oil if it is subjected to a pressure of 3000 kPa. (Assume that B = 1700 MPa.)

$$B = \frac{-P}{\Delta V / V} = \frac{-PV}{\Delta V}$$

$$\Delta V = \frac{-PV}{B} = \frac{-(3 \text{ x } 10^6 \text{Pa})(5 \text{ L})}{(1.70 \text{ x } 10^9 \text{Pa})}$$

Decrease in V; milliliters (mL):

ΔV = -8.82 mL

Summary: Elastic and Inelastic

An **<u>elastic body</u>** is one that returns to its original shape after a deformation.

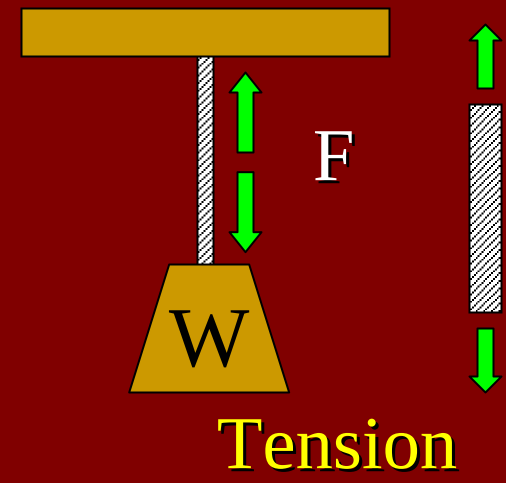
An elastic collision loses no energy. The deform-ation on collision is fully restored.

An **<u>inelastic body</u>** is one that does **<u>not</u>** return to its original shape after a deformation.

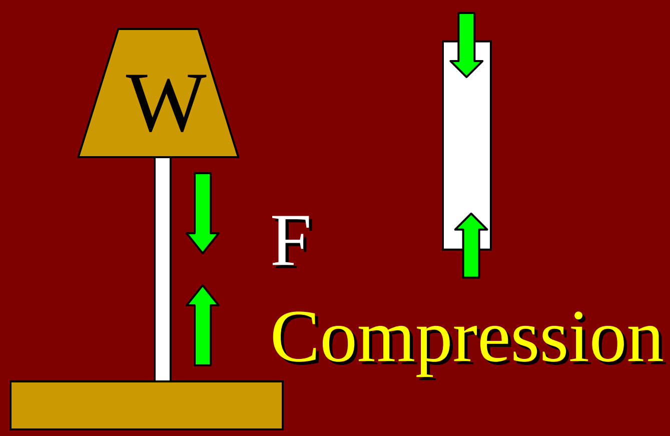
In an inelastic collision, energy is lost and the deformation <u>may</u> be permanent.

Summary
Types of Stress

A tensile stress occurs when equal and opposite forces are directed away from each other.



A compressive stress occurs when equal and opposite forces are directed toward each other.



Summary of Definitions

Stress is the ratio of an applied force F to the area A over which it acts:

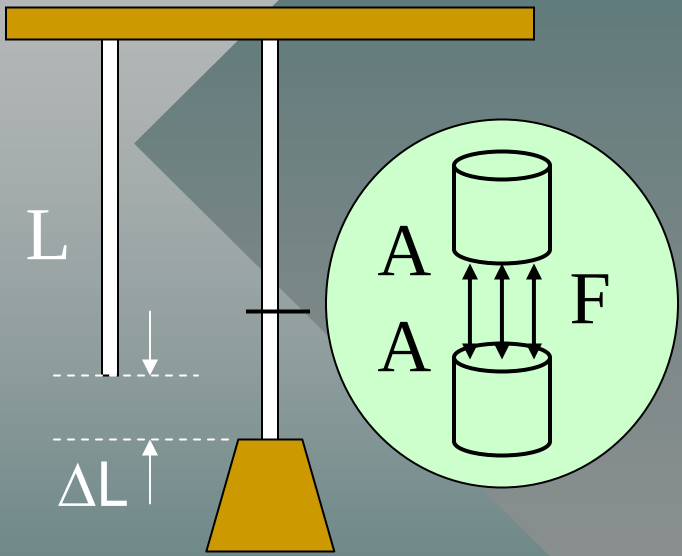
$$Stress = \frac{F}{A}$$

$$Units: \ \mathrm{Pa} = \frac{\mathrm{N}}{\mathrm{m}^2} \ \text{ or } \ \frac{\mathrm{lb}}{\mathrm{in.}^2}$$

Strain is the relative change in the dimensions or shape of a body as the result of an applied stress:

Examples: Change in length per unit length;
change in volume per unit volume.

Longitudinal Stress and Strain



For wires, rods, and bars, there is a longitudinal stress F/A that produces a change in length per unit length. In such cases:

$$Stress = \frac{F}{A}$$

$$Strain = \frac{\Delta L}{L}$$

The Elastic Limit

The elastic limit is the maximum stress a body can experience without becoming permanently deformed.

The Ultimate Strength

The ultimate strength is the greatest stress a body can experience without breaking or rupturing.

Young's Modulus

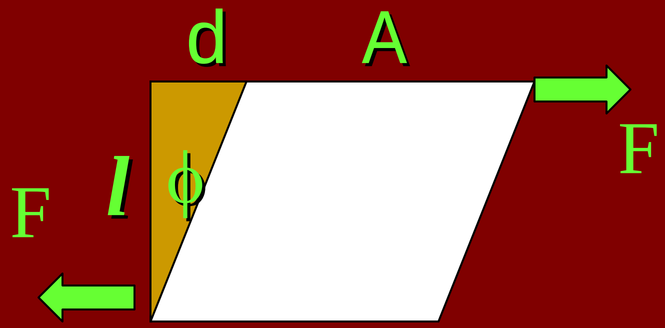
For materials whose length is much greater than the width or thickness, we are concerned with the longitudinal modulus of elasticity, or Young's Modulus Y.

$$Young's\ modulus = \frac{longitudinal\ stress}{longitudinal\ strain}$$

$$Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

$$Units:\ \text{Pa}\ \ or\ \ \frac{\text{lb}}{\text{in.}^2}$$

The Shear Modulus



Stress is force per unit area:

$$Stress = \frac{F}{A}$$

The strain is the angle expressed in <u>radians</u>:

$$Strain = \phi = \frac{d}{l}$$

The shear modulus S is defined as the ratio of the shearing stress *F/A* to the shearing strain φ:

The shear modulus:
Units are in Pascals.

$$S = \frac{F/A}{\phi}$$

The Bulk Modulus

$$B = \frac{\text{Volume stress}}{\text{Volume strain}} = \frac{-F/A}{\Delta V/V}$$

Since *F/A* is generally pressure *P*, we may write:

$$B = \frac{-P}{\Delta V/V} = \frac{-PV}{\Delta V}$$

Units remain in Pascals (Pa) since the strain is unitless.

CONCLUSION:
Chapter 13 - Elasticity